**Exam Physics Laboratory 1: Data and error analysis** **25 November 2013**

This exam consists of **5 exercises** on 3 pages. Make each exercise on a separate sheet of paper! Write your name and student number on each sheet of paper! Write clearly, using a pen (not a pencil). A simple scientific calculator is allowed during the exam, but a graphing calculator is not permitted.

**Exercise 1** *(4 points)*

Rewrite the following results, using the correct notation:

a) $T = 312.659$ K $\pm$ 254 mK

b) $\lambda = 1.064$ $\mu$m $\pm$ 9.5 nm

c) $R = 47$ k$\Omega$ $\pm$ 33 $\Omega$

d) $p = 101.3$ kPa $\pm$ 16 Pa

**Exercise 2** *(5 points)*

The speed of sound $v$ in a gas is under certain conditions given by:

$$v = \sqrt{\frac{5p}{3\rho}}$$

with $p$ the pressure and $\rho$ the density of the gas. The pressure and density have been measured: $p = 1.01 \pm 0.01$ N/m$^2$ and $\rho = 1.21 \pm 0.02$ kg/m$^3$.

a) Calculate the speed of sound $v$.

b) Calculate the relative error and the absolute error in $v$.

c) Write the final result in the correct notation: $v = ... \pm ...$

**Exercise 3** *(9 points)*

Two independent measurements of the mass $M$ of an object yield: $M_1 = 18.9 \pm 0.3$ kg and $M_2 = 19.3 \pm 0.6$ kg.

a) Calculate the weighted average mass $M$.

b) Calculate the error $\Delta M$ of $M$.

c) If the mass is determined again using the same method, what is the probability of finding a value of $M \leq 18.8$ kg?

d) If the mass is determined again using the same method, what is the probability of finding a value of $M$ in the interval 19.1 - 19.2 kg?

| $z$ | $F(z)$ | $z$ | $F(z)$ | $z$ | $F(z)$ | $z$ | $F(z)$ |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 1.0 | 0.3413 | 2.0 | 0.4772 | 3.0 | 0.4987 |
| 0.1 | 0.0398 | 1.1 | 0.3643 | 2.1 | 0.4821 | 3.1 | 0.4990 |
| 0.2 | 0.0793 | 1.2 | 0.3849 | 2.2 | 0.4861 | 3.2 | 0.4993 |
| 0.3 | 0.1179 | 1.3 | 0.4032 | 2.3 | 0.4893 | 3.3 | 0.4995 |
| 0.4 | 0.1554 | 1.4 | 0.4192 | 2.4 | 0.4918 | 3.4 | 0.4997 |
| 0.5 | 0.1915 | 1.5 | 0.4332 | 2.5 | 0.4938 | 3.5 | 0.4998 |
| 0.6 | 0.2258 | 1.6 | 0.4452 | 2.6 | 0.4953 | 3.6 | 0.4998 |
| 0.7 | 0.2580 | 1.7 | 0.4554 | 2.7 | 0.4965 | 3.7 | 0.4999 |
| 0.8 | 0.2881 | 1.8 | 0.4641 | 2.8 | 0.4974 | 3.8 | 0.4999 |
| 0.9 | 0.3159 | 1.9 | 0.4713 | 2.9 | 0.4981 | 3.9 | 0.5000 |

Given:

$$N(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)$$

$$F(z) = \int_0^z N(y)dy$$

**Exercise 4** *(6 points)*

The resistance $t$ of a pendulum is measured 5 times, with the following results:
$t = 6.17$ s, 6.13 s, 6.23 s, 6.11 s, 6.16 s.

a) Calculate the best estimate for the average period of the pendulum.

b) Calculate the best estimate for the standard deviation $\sigma$ of these measurements.

c) Calculate the error in the best estimate for the period calculated in part a).

**Exercise 5** *(12 points)*

The following formulae are given for fitting data to a straight line $y = ax + b$:

$$a = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2},$$

$$b = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2},$$

$$(\Delta a)^2 = \left(\frac{1}{\sum x_i^2 - N\overline{x}^2}\right) \frac{\sum r_i^2}{N-2},$$

$$(\Delta b)^2 = \left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2}\right) \frac{\sum r_i^2}{N-2}.$$

The resistance $R$ of a platinum wire is sometimes used as a thermometer. Based on theory, it is known that $R$ is a function of temperature $T$:

$$R(T) = R_0 + \alpha T$$

with $R_0$ and $\alpha$ constants. For a specific platinum wire the following measurement results have been obtained:

| $T$(°C) | $R$ (Ω) |
|---|---|
| -50.0 | 80.3 |
| -40.0 | 84.0 |
| -30.0 | 88.3 |
| -20.0 | 92.0 |
| -10.0 | 96.3 |

The error in $T$ is negligible.

a) Calculate the best estimates for the constants $R_0$ and $\alpha$ using the method of least squares.

b) Calculate the errors in $R_0$ and $\alpha$.

c) Calculate the value $R$ and its error $\Delta R$ at a temperature $T = -25.5$°C (assume in this case $\Delta T = 0.1$°C).

d) A smart student performs a fit to a parabolic function $R = pT^2 + qT + w$ as well and finds $\sum r_i^2 = 0.0823$, where $r_i$ is the difference between observed value and fitted value. Calculate $\chi^2$ (necessary for the $\chi^2$ test). Assume $\Delta R = 0.5\ \Omega$.

e) Suppose the 10% - 90% probability level is chosen. Using the table below, indicate whether a parabolic function is acceptable as well or not.

| $F =$ / $\nu$ | 0.01 | 0.10 | 0.50 | 0.90 | 0.99 |
|---|---|---|---|---|---|
| 1 | 0.000 | 0.016 | 0.455 | 2.706 | 6.635 |
| 2 | 0.020 | 0.211 | 1.386 | 4.605 | 9.210 |
| 3 | 0.115 | 0.584 | 2.366 | 6.251 | 11.35 |
| 4 | 0.297 | 1.064 | 3.357 | 7.779 | 13.28 |
| 5 | 0.554 | 1.610 | 4.351 | 9.236 | 15.09 |

Table 1: Cumulative $\chi^2$ distribution $F(\chi^2|\nu)$.

*Exam grade = (total of points) / 4 + 1*